

STAT 706 Homework 5
SPRING 2019

Due on Tuesday March 12 at the beginning of lecture. Note: R stands for exercises in Rosenthal's book.

1. (R 4.5.14 (b))

Let Z_1, Z_2, \dots be general random variables with $E[|Z_i|] < \infty$, and let $Z = Z_1 + Z_2 + \dots$. Suppose that at least one of $\sum_i E[Z_i^+] < \infty$ or $\sum_i E[Z_i^-] < \infty$. Prove that

$$E[Z] = \sum_i E[Z_i].$$

2. (R 5.5.2)

Give an example of a random variable X and constant $\alpha > 0$ such that

$$P(X \geq \alpha) > \frac{E[X]}{\alpha}.$$

Where does the proof of Markov's inequality break down in this case?

3. (R 5.5.3 (b))

Give an example of a random variable Y with $E[Y] = 0$ and $Var(Y) = 1$ such that

$$P(|Y| \geq 2) < 1/4.$$

4. (R 5.5.12)

Give an example of two discrete random variables having the same mean and the same variance, but which are not identically distributed.

5. (R 5.5.14)

Prove the converse of Lemma 5.2.1. That is, prove that if $\{X_n\}$ converges to X almost surely, then for each $\epsilon > 0$ we have $P(|X_n - X| \geq \epsilon \text{ i.o.}) = 0$.

6. (R 5.5.15)

Let X_1, X_2, \dots be a sequence of independent RVs with $P(X_n = 3^n) = P(X_n = -3^n) = \frac{1}{2}$. Let $S_n = X_1 + \dots + X_n$.

(a) Compute $E[X_n]$ for each n .

(b) For $n \in \mathbb{N}$, compute $R_n \equiv \sup\{r \in \mathbb{R} : P(|S_n| \geq r) = 1\}$.

(c) Compute $\lim_{n \rightarrow \infty} \frac{1}{n} R_n$.

(d) For which $\epsilon > 0$ (if any) is it the case that $P(\frac{1}{n}|S_n| \geq \epsilon) \not\rightarrow 0$?

(e) Why does this result not contradict the various laws of large numbers?